Notes

$$
\begin{aligned}
T &- \text{sampling time} \\
\omega_c &- \text{cutoff frequency} \\
\omega_r &- \text{resonance frequency} \\
\omega_l &- \text{lower band width limit} \\
\omega_h &- \text{higher band width limit}
\end{aligned}
$$

Discrete forms use Euler derivative ($s \to \frac{1-z^{-1}}{T}$)

Low pass

$$
\begin{aligned}
\omega_c &= \frac{1}{RC} \\
H_{LP}(s) &= \frac{1}{RCs+1} = \frac{1}{\frac{1}{\omega_c}s+1} \\
H_{LP}(z) &= \frac{\frac{T}{RC}}{[\frac{T}{RC}+1]-z^{-1}} = \frac{\omega_c T}{[\omega_c T+1]-z^{-1}}
\end{aligned}
$$

High pass

$$
\begin{aligned}
H_{HP}(s) &= 1 - H_{LP}(s) \\
H_{HP}(s) &= \frac{RCs}{RCs+1} = \frac{\frac{1}{\omega_c}s}{\frac{1}{\omega_c}s+1} \\
H_{HP}(z) &= \frac{1-z^{-1}}{[\frac{T}{RC}+1]-z^{-1}} = \frac{1-z^{-1}}{[\omega_c T+1]-z^{-1}}
\end{aligned}
$$

Band pass

$$
\begin{aligned}
\omega_r &= \frac{1}{\sqrt{LC}} = \sqrt{\omega_l \omega_h} \\
B &= \frac{R^2 C}{L} = \frac{\omega_h}{\omega_l} \\
H_{BP}(s) &= \frac{RCs}{LCs^2+RCs+1} = \frac{\frac{\sqrt{B}}{\omega_r}s}{\frac{1}{\omega_r^2}s^2+\frac{\sqrt{B}}{\omega_r}s+1} \\
H_{BP}(z) &= \frac{1-z^{-1}}{[\frac{L}{RT}+\frac{T}{RC}+1]-[\frac{2L}{RT}+1]z^{-1}+\frac{L}{RT}z^{-2}} \\
&= \frac{1-z^{-1}}{[\frac{1}{\omega_r T\sqrt{B}}+\frac{T}{\omega_r\sqrt{B}}+1]-[\frac{2}{\omega_r T\sqrt{B}}+1]z^{-1}+\frac{1}{\omega_r T\sqrt{B}}z^{-2}}
\end{aligned}
$$

Band stop / Notch

$$
\begin{aligned}
H_{BS}(s) &= 1 - H_{BP}(s) \\
H_{BS}(s) &= \frac{LCs^2+1}{LCs^2+RCs+1} = \frac{\frac{1}{\omega_r^2}s^2+1}{\frac{1}{\omega_r^2}s^2+\frac{\sqrt{B}}{\omega_r}s+1} \\
H_{BS}(z) &= \frac{[\frac{T^2}{LC}+1]-2z^{-1}+z^{-2}}{[\frac{T^2}{LC}+\frac{RT}{L}+1]-[\frac{RT}{L}+2]z^{-1}+z^{-2}} \\
&= \frac{[\omega_r^2 T^2+1]-2z^{-1}+z^{-2}}{[\omega_r^2 T^2+\omega_r T\sqrt{B}+1]-[\omega_r T\sqrt{B}+2]z^{-1}+z^{-2}}
\end{aligned}
$$

MATLAB-Test

```
T = 0.001;
wc = 10;
wl = 1;
wh = 100;

disp('Low pass')

RC = 1/wc;

HLPs1 = tf(1, [RC, 1]) %#ok<*NOPTS>
HLPs2 = tf(1, [1/wc, 1]) %#ok<*NASGU>

HLPz1 = tf([T/RC, 0], [T/RC+1, -1], T)
HLPz2 = tf([wc*T, 0], [wc*T+1, -1], T)

disp('High pass')

HHPs1 = 1-HLPs1
HHPs2 = tf([RC, 0], [RC, 1])
HHPs3 = tf([RC, 0], [1/wc, 1])

HHPz1 = 1-HLPz1
HHPz2 = tf([1, -1], [T/RC+1, -1], T)
HHPz3 = tf([1, -1], [wc*T+1, -1], T)

disp('Band pass')

wr = sqrt(wl*wh);
B = wh/wl;
L = 0.123456;
C = 1/L/wr^2;
R = sqrt(B*L/C);

HBPs1 = tf([R*C, 0], [L*C, R*C, 1])
HBPs2 = tf([sqrt(B)/wr, 0], [1/wr^2, sqrt(B)/wr, 1])

HBPz1 = tf([1, -1, 0], [L/R/T+T/R/C+1, -(2*L/R/T+1), L/R/T], T)
HBPz2 = tf([1, -1, 0], ...
    [1/wr/T/sqrt(B)+T/wr/sqrt(B)+1, -(2/wr/T/sqrt(B)+1), 1/wr/T/sqrt(B)], T)

disp('Band stop')

HBSs1 = 1-HBPs1
HBSs2 = tf([L*C, 0, 1], [L*C, R*C, 1])
HBSs3 = tf([1/wr^2, 0, 1], [1/wr^2, sqrt(B)/wr, 1])

HBSz1 = 1-HBPz1
HBSz2 = tf([T^2/L/C+1, -2, 1], [T^2/L/C+R*T/L+1, -(R*T/L+2), 1], T)
HBSz3 = tf([wr^2*T^2+1, -2, 1], ...
    [wr^2*T^2+wr*T*sqrt(B)+1, -(wr*T*sqrt(B)+2), 1], T)

figure('name', 'Low Pass and High Pass')
hold('on')
bode(HHPs1)
bode(HLPs1)
bode(HLPz1)
bode(HHPz1)

figure('name', 'Band Pass and Band Stop')
hold('on')
bode(HBPs1)
bode(HBPz1)
bode(HBSs1)
bode(HBSz1)
chld = get(gcf, 'children');
set(chld(3), 'ylim', [-50, 0]);
```